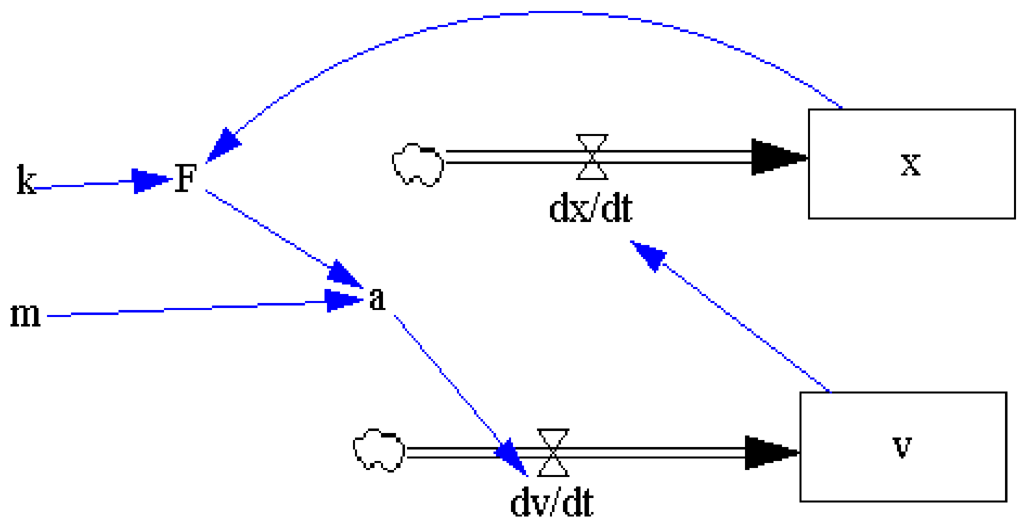
❖ V12 – zadatak i skica modela

- Koristeći Vensim napravite simulaciju titranja tijela mase m = 20g obješenog o oprugu konstante elastičnosti k = 80 mN m^{-1} ako se tijelo u početnom trenutku nalazi 2 cm od ravnotežnog položaja i ima brzinu -0.5 cm s^{-1}.
  - ✓ rješenje: 08_V12.mdl
  - ✓ rješavamo analogno kao prethodni problem

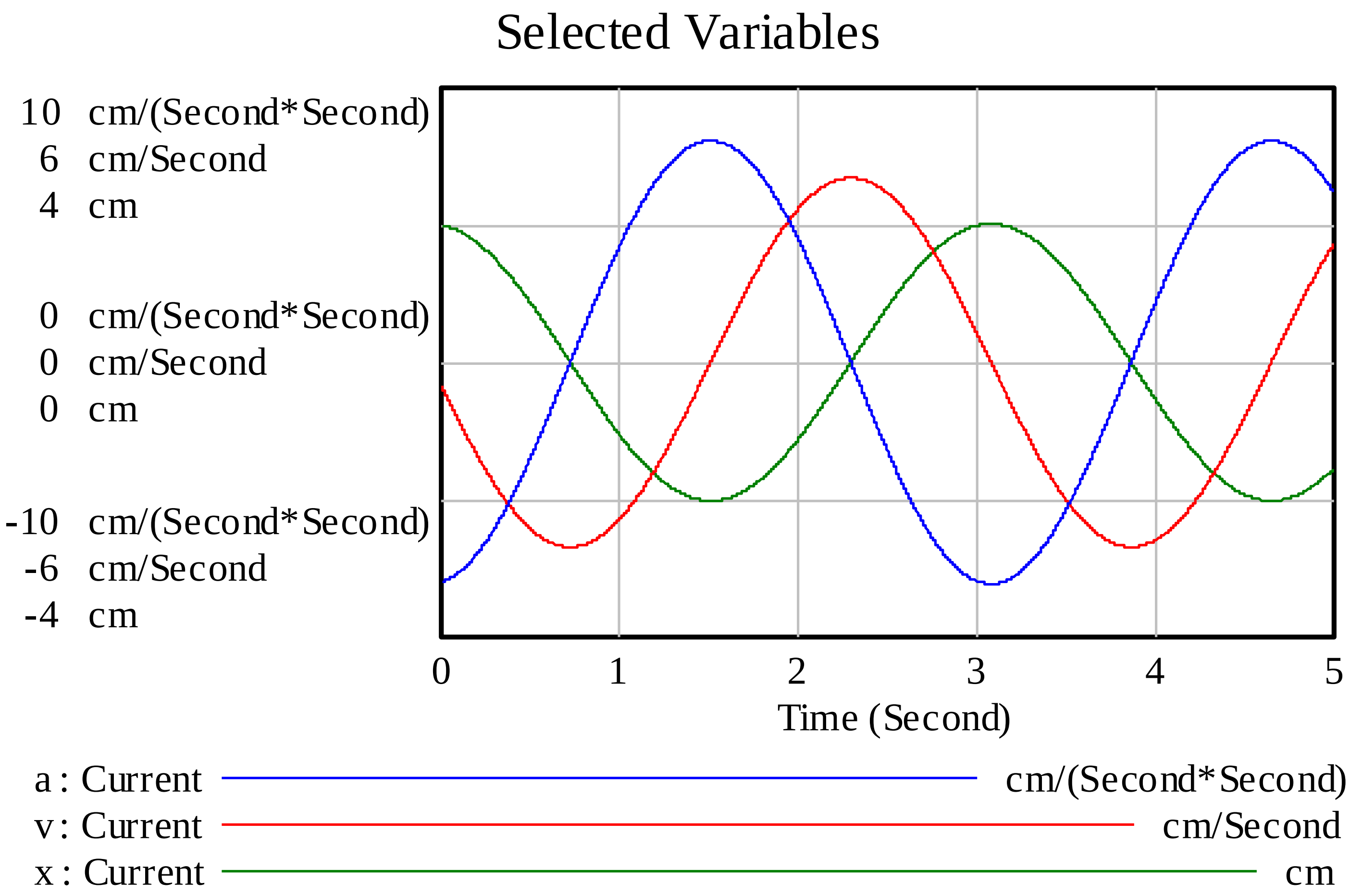
$$F = -kx \;;\; a = \frac{F}{m} \;;\; \frac{dv}{dt} = a \;;\; \frac{dx}{dt} = v$$

$$x(0) = 2\ \text{cm};\quad \text{v}(0) = -0.5\text{cm/s}$$



❖ V12 - rezultati

✓ ovisnost položaja, brzine i akceleracije o vremenu



✓ egzaktno rješenje $x(t) = A\cos(\omega_0 t + \varphi)$

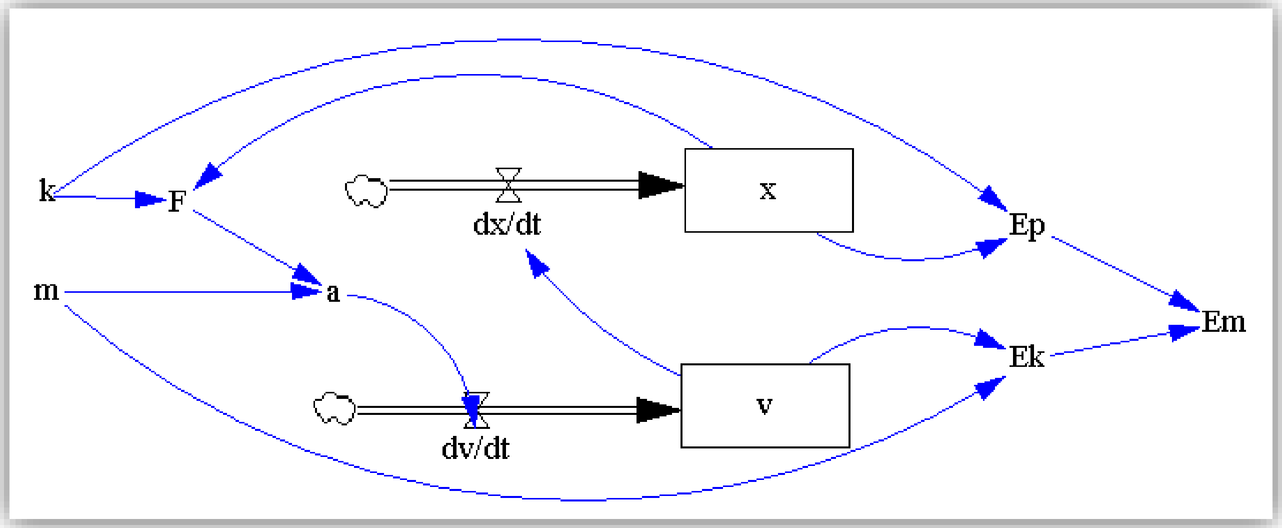
$$\omega_0 = \sqrt{k/m}$$

❖ V13 – zadatak i skica modela

- Proširite zadatak V12:
  - a) tako da računa kinetičku, potencijalnu i mehaničku energiju za linearni harmonijski oscilator;
  - b) uračunajte i silu trenja te promotrite kako se mijenjaju oscilacije povećavanjem proizvoljno odabrane konstante gušenja;
  - c) kako dodavanje vanjske sile F(t)=0.2Ncos(ωt) i iznos njene kutne frekvencije ω utječe na oscilacije

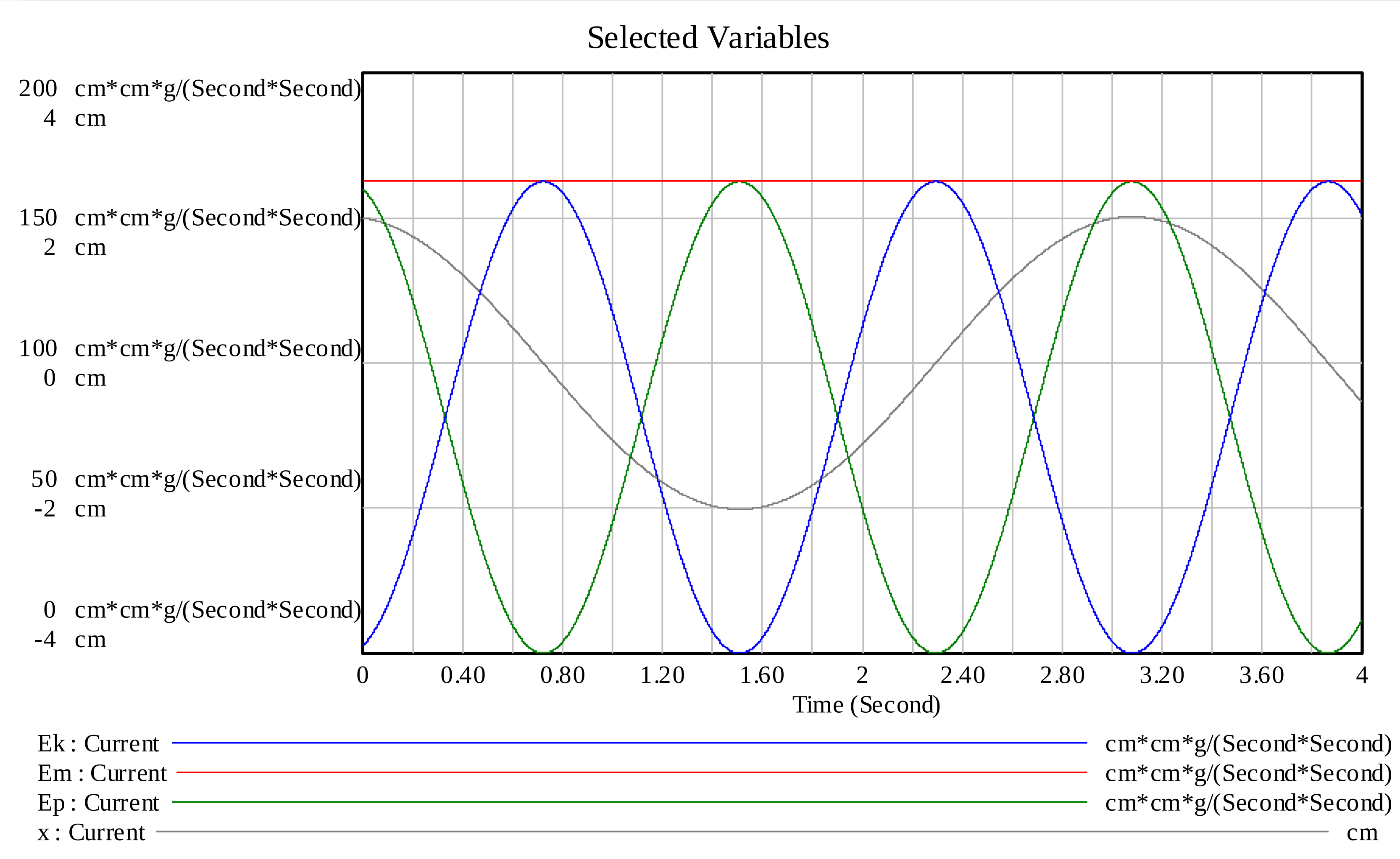
a) 08_V13a.mdl rješavamo analogno kao prethodni problem dodajući

$$E_k = \frac{1}{2}mv^2 \;;\quad E_p = \frac{1}{2}kx^2 \quad;\quad E_m = E_k + E_p$$



❖ V13a - rezultati

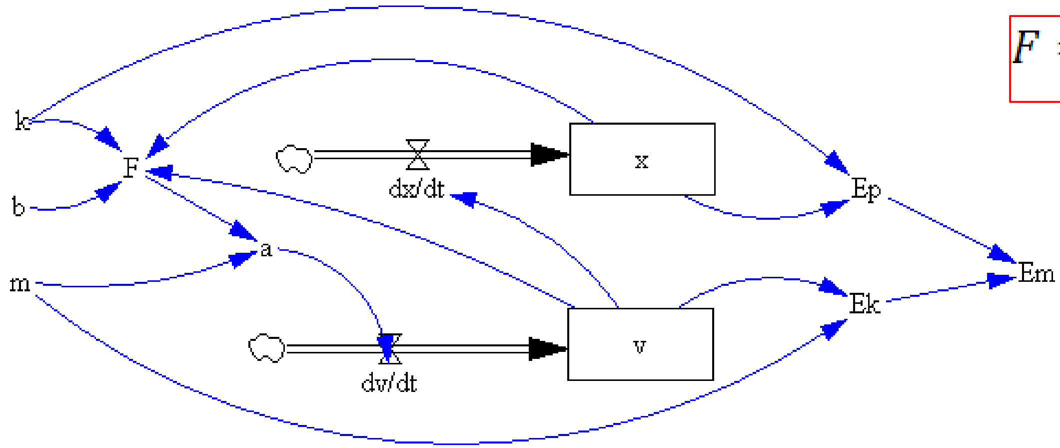
a)



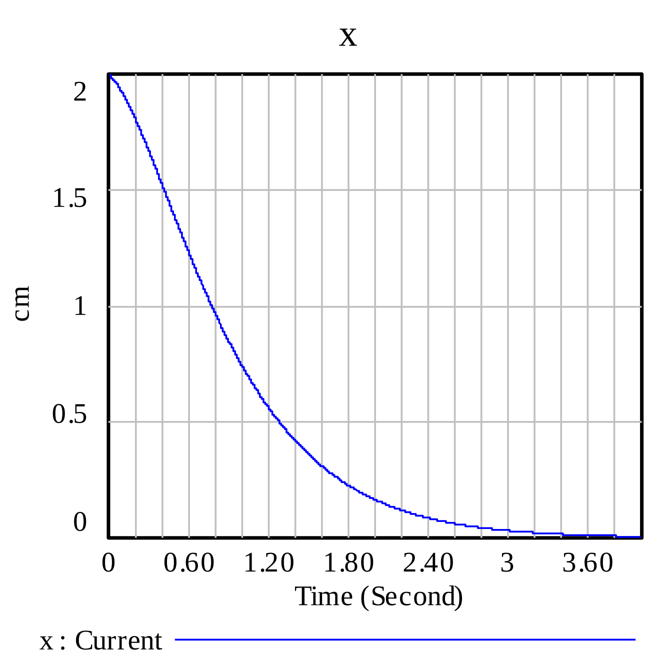
❖ V13b – kritično prigušenje

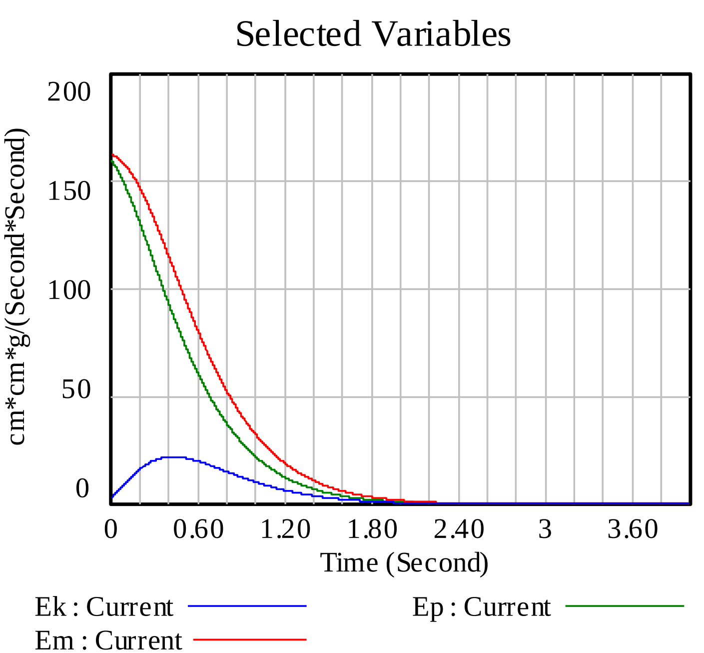
b) dodamo koeficijent gušenja b i silu otpora –bv (08_V13b.mdl)

$$F = -kx - bv$$



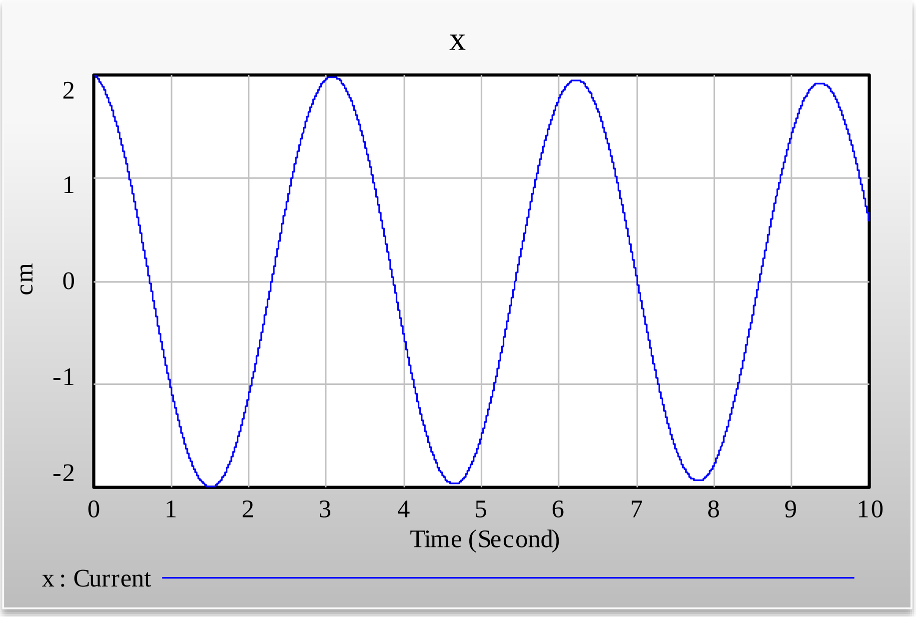
❖ pr. *b* = 80g/s => kritično prigušenje: $b = \sqrt{4km} = 80\text{gs}^{-1}$

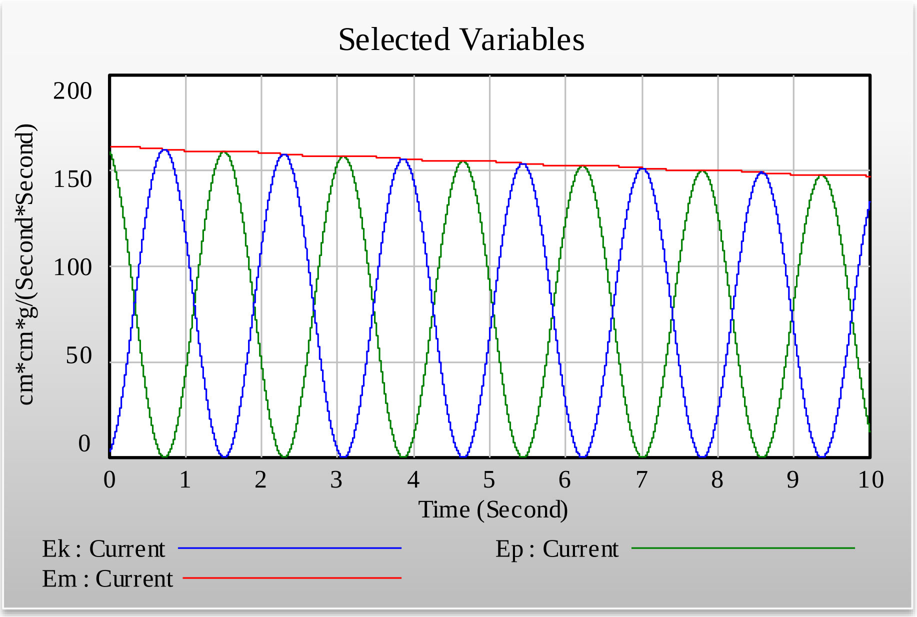




❖ V13b – malo prigušenje

❖ pr. b = 0.2 g/s => malo prigušenje: $\left(\sqrt{4km} = 80\mathrm{gs}^{-1}\right) > \left(b = 0.2\mathrm{gs}^{-1}\right)$





❖ egzaktno rješenje $x(t) = A\exp(-\delta t)\cos(\omega t + \varphi)$

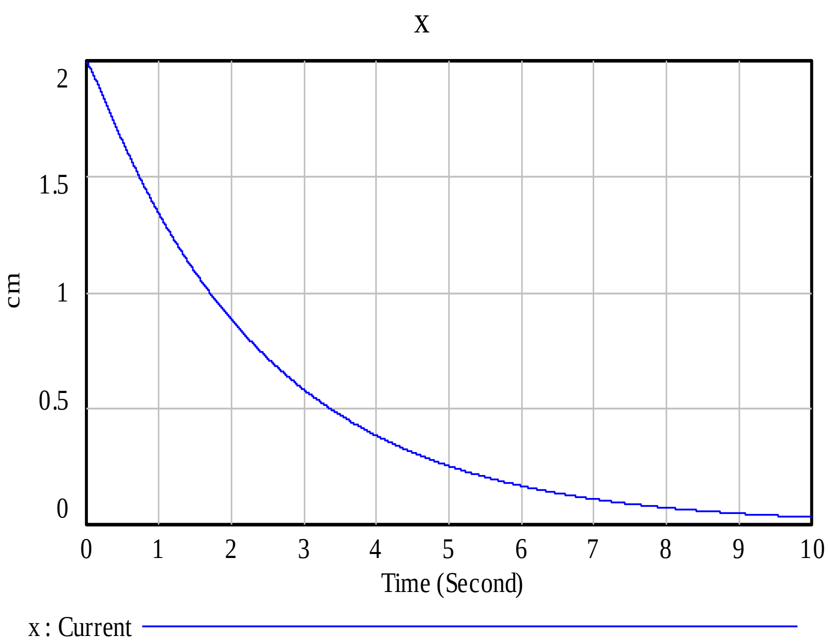
$$\omega = \sqrt{\omega_0^2 - \delta^2}$$

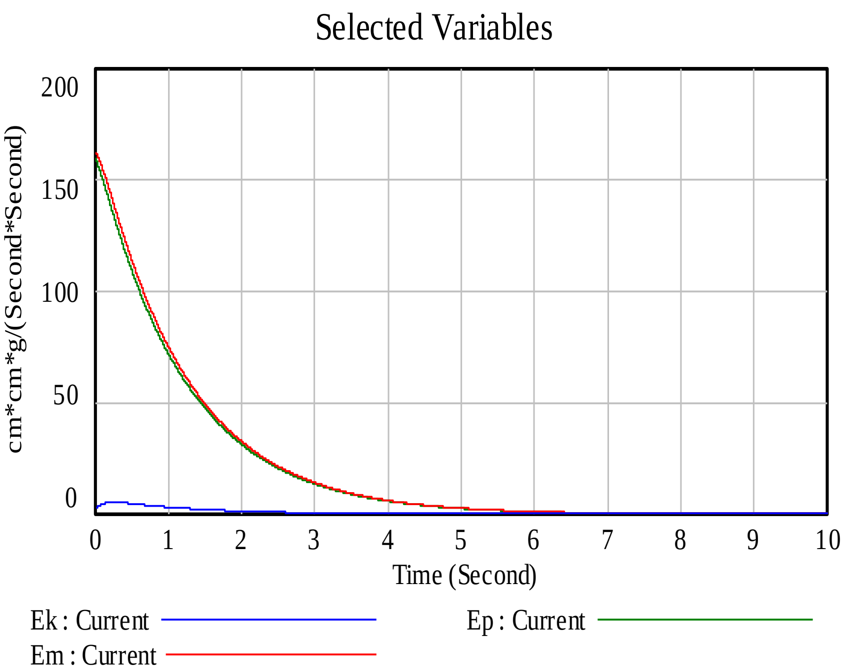
$$\omega_0 = \sqrt{k/m} \qquad \delta = \frac{b}{2m}$$

❖ V13b – aperiodično prigušenje

- pr. b = 200 g/s => aperiodično prigušenje:

$$\left(\sqrt{4km} = 80\mathrm{gs}^{-1}\right) < \left(b = 200\mathrm{gs}^{-1}\right)$$



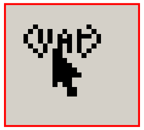


❖ V13c – prisilne oscilacije

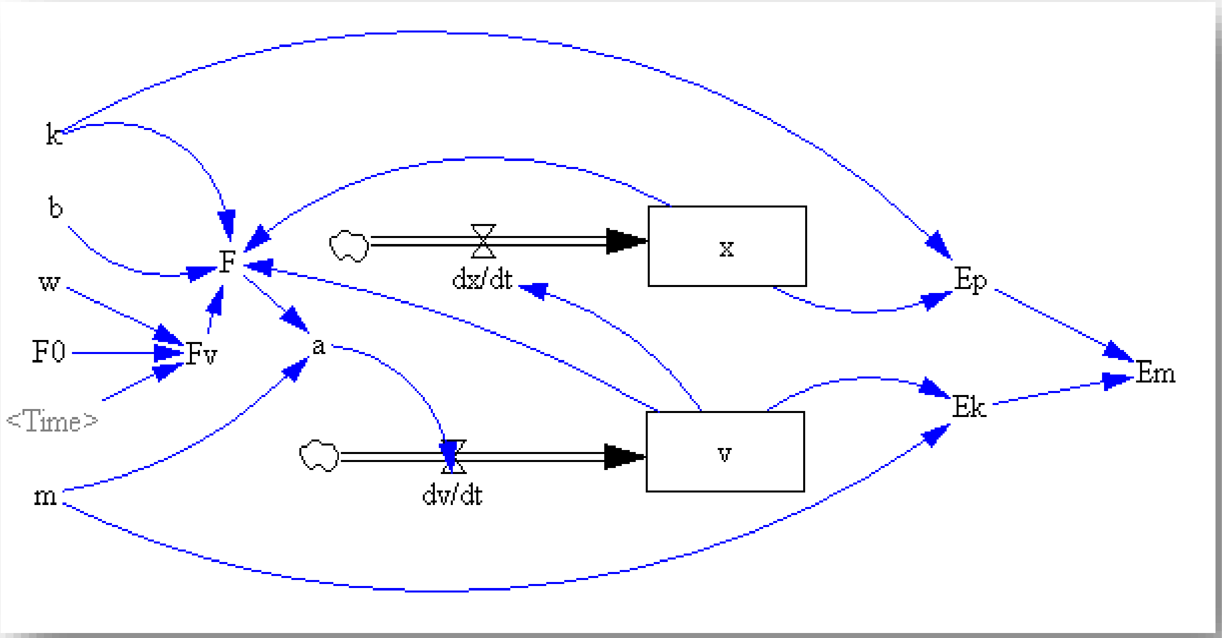
c) dodamo još i vanjsku oscilirajuću silu *F*v (08 V13c.mdl)

$$F = -kx - bv + F_0 cos(\omega t)$$

❖ objekt, koji samo čita vrijednost neke varijable, kao npr. vrijeme dodajemo koristeći varijable sjene



❖ vrijeme = <Time>



❖ V13c – rezonantni odaziv za $\omega \approx \omega_0$

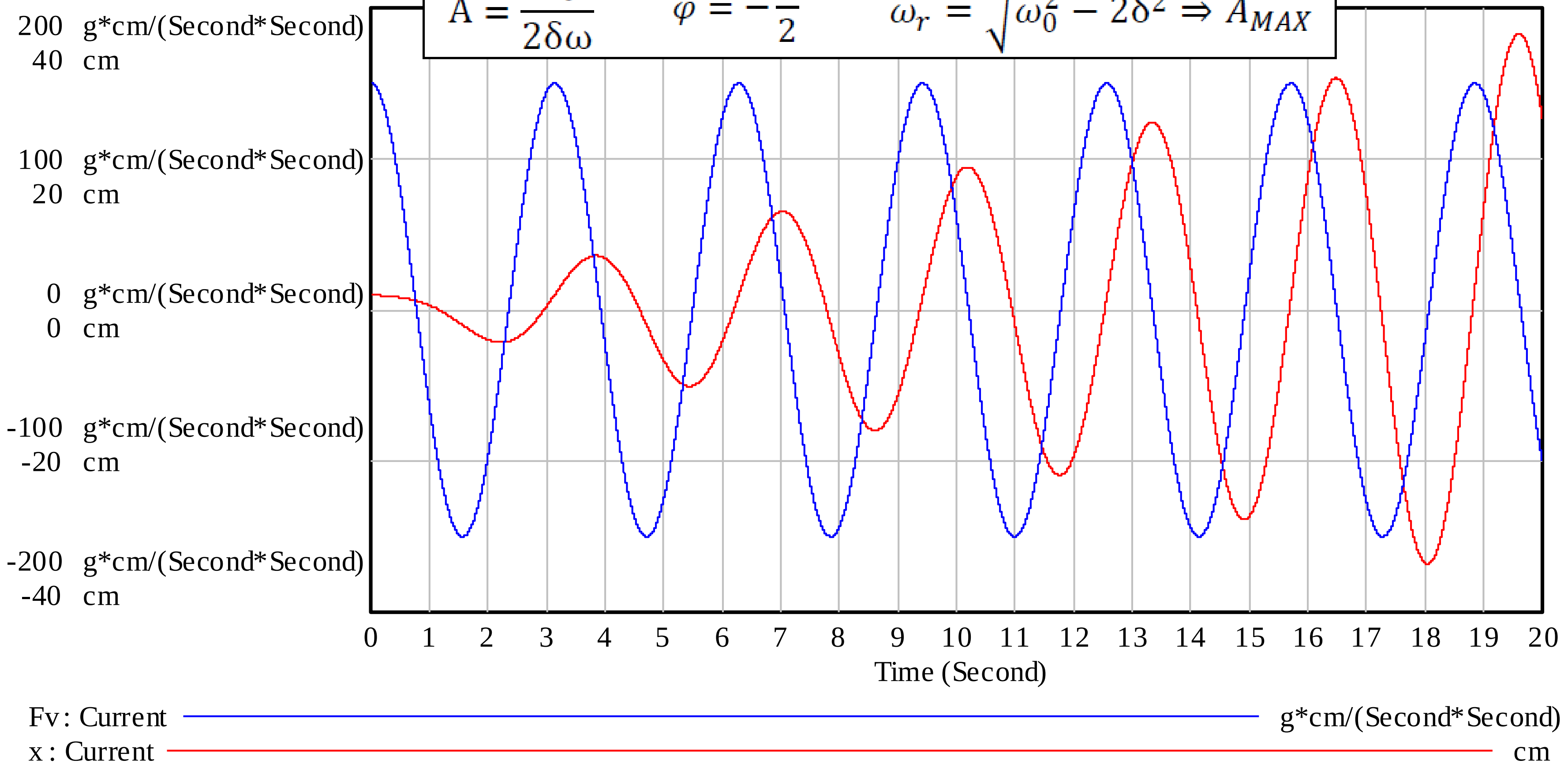
c) opruga određuje odaziv, a ne masa ili otpor

k = 80 gs^{-2}; b = 0 gs^{-1}; m = 20 g; F_0 = 1.5 mN

- ❖ ω = 2s^{-1}
- ❖ sustav kasni za vanjskim oscilatorom za četvrtinu perioda

$$\omega \approx \omega_0$$

$$A = \frac{A_0}{2\delta\omega} \qquad \varphi = -\frac{\pi}{2} \qquad \omega_r = \sqrt{\omega_0^2 - 2\delta^2} \Rightarrow A_{MAX}$$



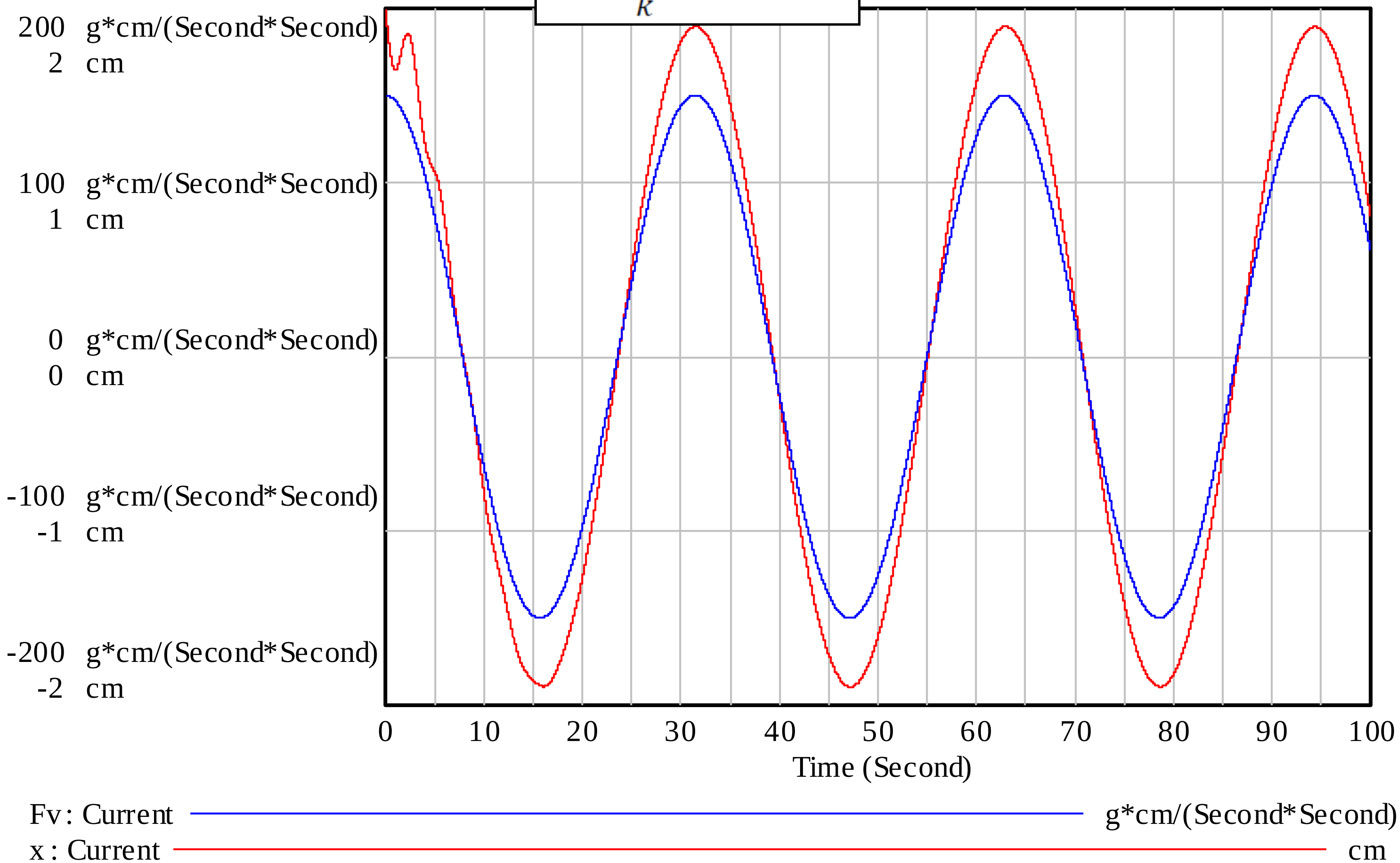
❖ V13c – rezultati za $\omega \ll \omega_0$

c)

- $\omega = 0.2 s^{-1}$
- odziv sustava je u fazi s pogonskom silom

$$\omega \ll \omega_0$$

$$A = \frac{F_0}{k} \qquad \varphi = 0$$



❖ V13c – rezultati za $\omega >> \omega_0$

c)

- Inercija tijela ograničava odaziv; $\omega = 7s^{-1}$
- sustav kasni za vanjskim oscilatorom za pola perioda
- inercija tijela ograničava odziv

$$\omega \gg \omega_0$$

$$A = \frac{A_0}{\omega^2} \qquad \varphi = -\pi$$